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Experimental Tests of the Holographic Entropy Bound

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Kovtun, Son and Starinets proposed a bound on the viscosity of any fluid in terms of its entropy density. The bound is saturated by maximally supersymmetric theories at strong coupling, but can also easily be challenged experimentally to within a factor of 10 already today. We argue that this bound follows directly from the generalized covariant entropy bound, bringing holography within the reach of experimental investigation.

I. INTRODUCTION

Kovtun, Son and Starinets recently proposed [1] that in nature the value[12]

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B} \approx 6.08 \times 10^{-13} K \cdot s, \quad (1)$$

where η is the shear viscosity of a fluid and s its entropy density, plays a fundamental role. They showed that idealized systems, such as strongly coupled $\mathcal{N} = 4$ supersymmetric Yang-Mills theory and similar theories in 3 and 6 dimensions precisely realize this value, while they conjectured that in general it provides a *lower* bound. One can easily envision that this hypothesis could be tested experimentally in the near future. While for example water under normal conditions (298.15 K, atmospheric pressure) has a value of $\frac{\eta}{s} = 2.3 \times 10^{-10} Ks$, for superfluid helium ^4He one can obtain values down to $\frac{\eta}{s} = 6.20 \times 10^{-12} Ks$, only a factor of 10 away from the bound [1, 2].

What we will argue in this paper is that the viscosity bound can be thought of as a consequence of the holographic entropy bound, or more precisely from the generalized covariant entropy bound (GCEB) of [3]. Holographic entropy bounds are believed to be a generic and fundamental property of any theory of quantum gravity, (see [4, 5] or [6] for a review). Roughly speaking, the statement is that the number of degrees of freedom in the universe don't scale as the volume of the universe, but only as its surface area. A precise version of this hypothesis, the Covariant Entropy Bound (CEB), has been formulated by Bousso [7]. The GCEB is an even stronger version of this bound.

So far all these bounds have been motivated by theoretical necessity. While many accept the CEB, the status of the GCEB even among believers is somewhat less clear. Experimental tests of the entropy bounds are in principle possible[13]. But one runs into the usual problem that current experiments are orders of magnitude away from testing quantum gravity. Showing that the GCEB implies the viscosity bound of [1] would bring testing holog-

raphy into experimental reach. We make the first steps into this direction.

II. HYDRODYNAMICS AND VISCOSITY

To understand the statement of the bound, we have to briefly introduce the basic notions of hydrodynamics and how it relates to strongly coupled systems with, following the nice discussion in [8]. Hydrodynamics should be viewed as an effective theory, describing small fluctuations around equilibrium of a thermal system on length and time scales which are large compared to any microscopic scale in the system. The relevant hydrodynamic degrees of freedom are the conserved charge densities, in the simplest case just for the charges derived from the energy-momentum tensor, $\epsilon \equiv T^{00}$ and $\pi^i \equiv T^{i0}$. One equation of motion is just the current conservation equation. In addition, one writes down the so called constitutive relations, expressing the fluxes of the conserved quantities (the spatial parts of the conserved currents) in terms of the hydrodynamic degrees of freedom. Without any further input, one has to allow here the most general set of terms consistent with symmetries. Since hydrodynamics is supposed to be an effective theory of long distances and small fluctuations, one can systematically perform a double expansion in powers of the fields and numbers of derivatives. To linear order in the fluctuations and to first order in derivatives, the constitutive relation for the spatial components of the stress-energy tensor in D spacetime dimensions reads

$$T^{ij} = \delta^{ij} (P + v_s^2 \delta\epsilon) - \gamma_\zeta \delta^{ij} \vec{\nabla} \cdot \vec{\pi} - \gamma_\eta \left(\nabla^i \pi^j + \nabla^j \pi^i - \frac{2}{D-1} \vec{\nabla} \cdot \vec{\pi} \right). \quad (2)$$

P is the equilibrium pressure. $\delta\epsilon$ denotes the fluctuation in T^{00} around the equilibrium value. For an equilibrium system in its rest frame, the momentum densities are zero, so that the π 's are already the first order fluctuations. The three transport coefficients, v_s^2 , γ_η and γ_ζ depend on the microscopic details of the theory. The terms linear in the momentum densities have been split into two independent tensor structures. The speed of sound v_s and the bulk viscosity coefficient γ_ζ govern the propagation and diffusion of longitudinal momentum

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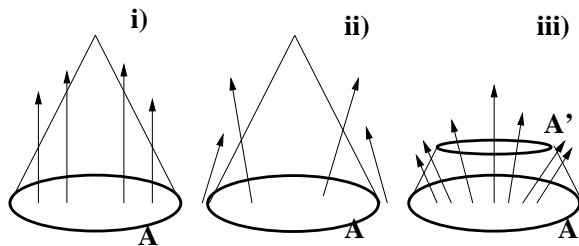


FIG. 1: i) illustrates the CEB: all matter contained within the area A has to pass the lightsheet, and this is still true even if there are velocities and velocity gradients as in ii). However in the case of the GCEB, where we are only interested in entropy passing through the lightsheet from A to A' , matter inside both A and A' , that would not have been counted at rest can move into the lightsheet as depicted in iii). It is only in this case that velocity gradients give new challenges to the bound.

fluctuations which form a coupled system with $\delta\epsilon$, while the shear viscosity coefficient γ_η governs the diffusion of transverse fluctuations with $\delta\epsilon = \vec{\nabla} \cdot \vec{\pi} = 0$, which to this order decouple from the sound waves. Conventionally one refers to the corresponding quantities $\eta = (\epsilon + P)\gamma_\eta$ and $\zeta = (\epsilon + P)\gamma_\zeta$ as the bulk and shear viscosity respectively. They would appear directly in the constitutive relation if we were to linearize in velocities instead of momenta. In a conformal theory like maximally supersymmetric Yang Mills theory, the stress-energy tensor has to be traceless, implying that $\zeta = 0$ and $v_s = \frac{1}{\sqrt{D-1}}$. The beauty of the hydrodynamic description is that it applies to any system which acts like a fluid at long distances, even strongly coupled ones. For weak coupling η can be calculated perturbatively. For strongly coupled theories one has to find alternative techniques. As shown in [1], at weak coupling $\frac{\eta}{s} \gg 1$, so that the bound only gets tested by strongly coupled systems.

III. GENERAL STRATEGY FOR DERIVATION OF THE BOUND

In order to understand how viscosity could appear in holographic entropy bounds, let us briefly recapitulate how gravity enforces such bounds. If we had a fluid in flat space at rest with a finite entropy density, one could obviously violate the bound. The bound states that the entropy passing the lightsheet, which is constructed by emitting lightrays from the boundary of a given volume element inward, should be less than the area divided by $4G\hbar$. For a volume element of the fluid in flat space, the whole fluid enclosed in a given area passes the lightsheet, so that the entropy should scale with the volume by the assumption that we have a finite entropy density. The way gravity avoids this conflict is that it makes it inconsistent to have such a fluid in *flat* space. The energy stored in the fluid will curve space-time sufficiently to rescue the holographic bound.

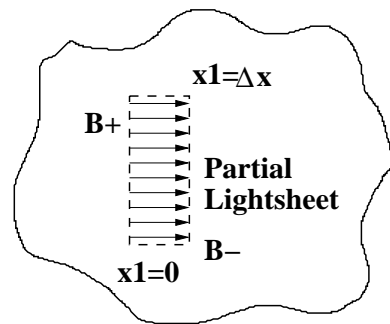


FIG. 2: A large droplet of fluid, out of which a small volume element gets sampled by a lightsheet starting at B_+ and terminated at B_- .

Viscosity becomes important, once the fluid has non-trivial velocity profiles. For the CEB, it does not seem to matter much whether the fluid is in motion: focus on a certain volume element of fluid. No matter what the velocities are, the whole fluid will eventually pass the lightsheet. On the other hand, no particles that were originally outside the volume can move inside fast enough to be counted by the lightsheet, since they are slower than light. The situation however is quite different for the GCEB. Here we are dealing with lightsheets that terminate, and the entropy is bounded by the area the lightsheet emanates from *minus* the area in which we chose to terminate it. Parts of the fluid that start out inside but sufficiently close to the spatial location at which we chose to terminate the lightsheet will be able to move outside the region before the light arrives there and avoid to be counted. Similar, fluid that starts outside the region we want to sample can move inside and might lead to a violation of the bound. This situation is sketched in Fig.1. As before, it is gravity that has to censor any possible violation. What we need is that velocity profiles lead to a curved spacetime, in which the lightsheet gets sufficiently modified. This is precisely where viscosity enters: from the constitutive relation we see that viscosity tell us how much stress T_{ij} we have in a fluid once we turned on the velocity profile. And by Einstein's equations this stress T_{ij} will turn on spacetime curvature R_{ij} . In order to not violate the bound, the backreaction has to be sufficiently large, that is for a fluid with given entropy density, the viscosity is not allowed to become too small. In the next section we will show how this works quantitatively.

IV. DERIVATION OF THE VISCOSITY BOUND

A. Review of Bousso's Derivation of Bekenstein's Bound

In [9] Bekenstein's bound on the entropy of a matter system was derived from the GCEB. The setup used there can easily be generalized to apply to the case of a fluid in motion. Consider a large region of spacetime

occupied by a fluid. We are interested in the entropy contained in a volume element bounded by surfaces B_+ and B_- . A lightsheet (a null-hypersurface generated by non-expanding light-rays) enters the volume element at B_+ and exits at B_- . The GCEB states that

$$S \leq \frac{A(B_+) - A(B_-)}{4G} \quad (3)$$

where S is the entropy passing through the lightsheet, and $A(B_i)$ denotes the area of the corresponding surface. For simplicity we will work with rectangular surfaces as depicted in Fig.2, but the result is independent of the shape, since the areas involved do not change if we deform B_\pm along the lightsheet. As in [9] we chose the lightsheet to be given by the set of parallel light rays obeying

$$x^0 = x^1; \quad (x^2, \dots, x^{D-1}) \text{ arbitrary constants.} \quad (4)$$

These are null geodesics in flat space. For the same reason that the area is invariant under deformations along the lightsheet we actually have $A(B_+) = A(B_-)$ and the entropy seems to be zero. In order to get a non-vanishing area difference we have to take into account the backreaction of the matter system. Due to the stress energy $T_{\mu\nu}$ of the fluid, non-trivial curvature $R_{\mu\nu}$ will be turned on, and the above hypersurface will neither be null nor geodesic. However, for weakly gravitating systems, there will be a close by null geodesic lightsheets. [9] uses two such close by lightsheets, one that starts in B_+ but misses B_- , and one that ends in B_- , but does not start at B_+ . Plugging those into eq.3, one obtains the following bound ($\hbar = 1$):

$$S \leq \pi \Delta x \int dx_2 \dots dx_D \int_0^{\Delta x} dx_1 T_{\mu\nu} k^\mu k^\nu, \quad (5)$$

where $k^{\mu\nu} = (1, 1, 0, 0)$ is a tangent null vector to the lightsheet. For the volume element of our fluid the rhs evaluates to

$$S_{eq.} = \pi V (\epsilon + P) \Delta x. \quad (6)$$

What will be important in this expression later is that it is proportional to Δx , so that the equilibrium configuration will lead to zero entropy in infinitesimally thin spatial slices. In [9] it was further observed that if one were to chose a lightsheet that includes the whole region of spacetime occupied by the fluid, $\int dx_2 \dots dx_D \int_0^{\Delta x} dx_1 T_{\mu\nu} k^\mu k^\nu$ just evaluates the total ADM mass of the fluid in its rest frame and one can recover the conventional form of Bekenstein's bound [10], $S \leq \pi M \Delta x$.

B. Letting Things Flow

In order for viscosity to enter the game we have to allow for motion of the fluid. Consider a fluid as described

above, but turn on a perturbation away from equilibrium which at $t = 0$ has the following momentum density profile:

$$\pi_1(\vec{x}, t = 0) = -\text{sgn}(x_1 - \Delta x/2) \cdot (\epsilon + P) \cdot v_0 \quad (7)$$

where ϵ and P are the equilibrium values of the energy density and the pressure.

$$v_0 \leq 1 \quad (8)$$

is the velocity at which the fluid is moving with respect to the restframe of the equilibrium system (which has to be less than the speed of light). Strictly speaking, as v_0 approaches 1 (or if we go out to large values of x_2), we would have to use relativistic hydrodynamics instead and, in addition, are no longer justified to neglect the higher order perturbations in the constitutive relation. So our derivation of the bound only applies to non-relativistic systems, even though we expect the bound to be also true in the relativistic case.

We have set up the system in such a way that the fluid at negative x_1 is moving to the right (in vane trying to escape the lightsheet), while the fluid at positive x_1 is moving to the left (rushing into the lightsheet). In addition we turn on

$$\pi_2(\vec{x}, t = 0) = 2\delta(x_1 - \Delta x/2) \cdot x_2 \cdot (\epsilon + P)v_0 \quad (9)$$

so that

$$\vec{\nabla} \cdot \vec{\pi} = \partial_1 \pi_1 + \partial_2 \pi_2 = 0 \quad (10)$$

and we are dealing with a purely transverse fluctuation. The linearized hydrodynamics can be solved in this case and the full time dependent solution is obtained by having the stepfunction diffuse with diffusion constant γ_η ,

$$\pi_1(\vec{x}, t) \sim \text{erf}\left(\frac{x - \Delta x/2}{\sqrt{4\gamma_\eta t}}\right). \quad (11)$$

In order to calculate the maximal entropy density allowed within the lightsheet bounded by $0 < x_1 < \Delta x$, we follow the same logic as in the derivation of Bekenstein's bound and find once more

$$s \leq \frac{\pi}{V} \int dx_2 \dots dx_D \Delta x \int_0^{\Delta x} dx_1 T_{ab} k^a k^b \quad (12)$$

$$= \pi \int_0^{\Delta x} dx_1 (T_{00} + 2T_{01} + T_{11}). \quad (13)$$

The equilibrium values on the right hand side as before yield $s_{eq.} = \pi \Delta x (\epsilon + P)$. In the same way, the contribution from T_{01} gives an integral over the momentum density π . The way we set up the profile and the lightsheet geometry, the net momentum in our lightsheet is actually zero. But even in a more general configuration, the T_{01} term will always give an integral over the local momentum density and vanish as we take Δx to zero,

which as we will soon see is all we need. Last but not least let's calculate what we get from the linear perturbation δT_{11} in the T_{11} piece. From the constitutive relation eq.2 we see that

$$\delta T_{11} = -2\gamma_\eta \partial_1 \pi_1(\vec{x}, t) \quad (14)$$

and on the right hand side we get a contribution

$$s \leq s_{eq.} + \pi 4v_0\eta \int_0^{\Delta x} dx_1 \delta(x_1 - \Delta x/2). \quad (15)$$

We have neglected the fact that the delta function diffuses while being sampled by the lightsheet. Since total momentum is conserved under diffusion, the integral over the full delta function gives an upper bound on the integral of the time dependent solution[14]

The important thing to note is that the viscosity term enters via an integral of a delta-function. We only get a surface contribution from the boundary of the lightsheet, the result is independent of Δx . Since the bound has to be satisfied for arbitrary lightsheets, it has to be true for lightsheets with infinitesimal Δx . In this case the bulk contributions, that is both the equilibrium contribution as well as the integral over the momentum density T_{01} , drop out[15]. Using further $v_0 \leq 1$ we obtain the viscosity bound of [1], $s \leq 4\pi\eta$.

V. INTERPRETATION AND FUTURE DIRECTION

Should one really interpret the viscosity bound as being some subtle imprint of quantum gravity on properties of macroscopic systems? For a somewhat more conservative point of view to read our results, recall the connection between Bekenstein's bound and the GCEB: the hypothetical holographic bound, involving both Newton's

constant G and Planck's constant \hbar gets combined with formulas of classical general relativity involving only G yielding a bound that only involves \hbar and turns out to basically correspond to Heisenberg's uncertainty principle. While one could read this as "Holography implies the uncertainty principle" a more conservative way to read this result is to say: once we chose to deal with quantum gravity, we obviously also have to treat the matter fields quantum mechanical, and the usual rules (including the uncertainty principle) will apply. Latter can be understood without ever appealing to quantum gravity.

In the same spirit we think one should read the viscosity bound. The GCEB can be used to derive a property of macroscopic systems, which presumably can also be derived by other methods. The reason it had not been obtained earlier is that it is a strong coupling phenomenon. As shown in [1], in a weakly coupled field theory, $\frac{\eta}{s} \gg 1$. The supersymmetric models saturating the bound are formally at infinite coupling, and the only reason one was able to compute $\frac{\eta}{s}$ is that they have a weakly coupled gravity dual. Still, the viscosity bound can be challenged experimentally, and any experimental test automatically becomes a test of the GCEB. For the future it might be very interesting to study in the same spirit non-vanishing chemical potentials or charged fluids in background fields. Most likely one can derive bounds similar to the viscosity bound from the GCEB that govern the behavior of other transport coefficients, giving even more opportunities to probe the GCEB in the laboratory.

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- [1] P. Kovtun, D. T. Son, and A. O. Starinets (2003), hep-th/0309213.
 - [2] *CRC handbook of chemistry and physics* (CRC press, 2002-2003).
 - [3] E. E. Flanagan, D. Marolf, and R. M. Wald, Phys. Rev. **D62**, 084035 (2000), hep-th/9908070.
 - [4] G. 't Hooft (1993), gr-qc/9310026.
 - [5] L. Susskind, J. Math. Phys. **36**, 6377 (1995), hep-th/9409089.
 - [6] R. Bousso, Rev. Mod. Phys. **74**, 825 (2002), hep-th/0203101.
 - [7] R. Bousso, JHEP **07**, 004 (1999), hep-th/9905177.
 - [8] P. Kovtun and L. G. Yaffe, Phys. Rev. **D68**, 025007 (2003), hep-th/0303010.
 - [9] R. Bousso, Phys. Rev. Lett. **90**, 121302 (2003), hep-th/0210295.
 - [10] J. D. Bekenstein, Phys. Rev. **D23**, 287 (1981).
 - [11] C. J. Hogan (2003), astro-ph/0310532.
 - [12] From now on we set $\hbar = k_B = 1$.
 - [13] For example it was argued in [11] that in principle the CEB implies a discreteness in the temperature variations of the cosmic microwave background.
 - [14] The diffusion process will also generate entropy which can be neglected as long as $\Delta t = \Delta x/c$ is much smaller than the diffusion timescale. See also the next footnote.
 - [15] Unfortunately for a realistic matter system, for which the fluid form of the stress tensor is only a long distance approximation, this limit seems to force us to take Δx so small that hydrodynamics is not necessarily a good description. One certainly can still setup a flow with the initial conditions we use, but to what extent it is the viscosity alone that governs the stress is up to debate.